The Weight of the Quantum

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Dedicated to the memory of Joy Rosenthal (1966-2023)
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Outline

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Quantum Theory and the Structure of Physics

2025 was the International Year of Quantum Science and Technology and the 100th birthday of quantum mechanics.
Quantum theory realizes the **atomic hypothesis** and it is true for everything we know about matter (Feynman!): biology, chemistry, CMP, molecular, atomic, nuclear and particle physics. (Frontiers: quantum biology, quantum gravity, quantum cosmology...)
Quantum physics is the basis of major technological advances that include semiconductors, modern electronics, lasers, atomic clocks, solar cells, medical imaging. (Frontiers: quantum information and computing, quantum cryptography, quantum sensors...)
Yet, quantum phenomena live in classical spacetime!! What is quantum ("atomic") spacetime? ("Ameres" of ancient atomists!)

Quantum Theory and the Structure of Physics

Main new points: Spacetime physics is classical, non-probabilistic, but background independent (generally covariant). Quantum physics is fundamentally probabilistic, with fixed Born rule. I will argue that **quantum gravity (QG) is"gravitized quantum theory (GQ)" of quantum spacetime with a dynamical and contextual Born rule and intrinsic triple and higher order quantum interference**. Formulae for the cosmological constant, Higgs mass, and masses and mixing matrices of quarks and leptons. **Predictions for neutrino masses**. QG=GQ: **metastrings (and metaparticles)** living in **quantum (modular) spacetime**. Dual particles - (fuzzy) dark matter. Dark energy - curvature of dual spacetime. **Dual spacetime ($[x, \tilde{x}] = i\lambda^2$) - the measurement problem, quantum vs classical and origin of the quantum. "Mock" quantum and complexity.**

Use the overall structure of physics to argue for "quantum gravity = gravitized quantum theory" (**QG=QG**) and **emphasize the need for empirical probes** such as **triple and higher-order quantum interference**, etc, at low energies. Our recent relevant papers on this topic:
[arXiv:2003.00318 [hep-th]], [arXiv:2202.06890 [hep-th]], [arXiv:2203.17137 [gr-qc]], [arXiv:2303.15645 [gr-qc]], [arXiv:2212.00901 [hep-th]], [arXiv:2212.06086 [hep-th]], [arXiv:2307.16712 [hep-th]], [arXiv:2407.06207 [hep-th]], [arXiv:2501.19269 [gr-qc]], [arXiv:2503.20854 [hep-th]].
Quantum theory emerges from compatibility of fundamental length/time (λ) and Lorentz symmetry. In general: "gravitized quantum theory". Quantum measurement and the quantum-classical transition come from averaging over the dual spacetime coordinates. New view on quantum fields.(CMP, HEP...). Mock quantum in complex systems.

But first: Quantum Mechanics is celebrating 100th birthday in 2025! **A very brief history** (Mehra and Rechenberg (9 books, 5000 pages); Duncan and Janssen (2 books, 1500 pages)): **1900–1913** (Kirchhoff, Wien, Lenard, Paschen, Rubens, Planck, Einstein, Ehrenfest, Nernst, Debye, Born, von Kármán...) and **1913–1923** (Balmer, Rydberg, Ritz, Rutherford, Geiger, Marsden, Haas, Nicholson, Bohr, Moseley, Barkla, Stark, Planck, Wilson, Ishiwara, Sommerfeld, Schwarzschild, Epstein, Einstein, Ehrenfest, Stern, Gerlach, Stoner, Pauli, Kronig, Goudsmit, Uhlenbeck, Ladenburg, van Vleck, Kramers, Landé...): *old quantum theory* (**Atomic models vs statistics) 1923-1927** (Heisenberg, Born, Jordan, Dirac, Pauli, Schrödinger, de Broglie, Bose, Einstein, Fermi, Lanczos, Eckart, Wiener, von Neumann, Kramers, Wentzel, Brillouin, Bohr...): *discovery of quantum mechanics*. **Quantum Field Theory (QFT) = special relativity and quantum theory. No experiment contradicts relativity and/or quantum theory!**

Brief history of quantization conditions:

Planck, Einstein: $E = nh\nu = n\hbar\omega$ (black body radiation; fluctuations imply wave-particle duality from Einstein to de Broglie)
Adiabatic theorem (Botzmann, Ehrenfest) for the harmonic oscillator $J = \int pdq = const$ implies $E/\omega = const$ $(= n\hbar)$
Bohr-Sommerfeld (and Planck, Wilson, Ishiwara, Schwarzschild, Epstein) $J_i = \int p_i dq_i = n_i h$; $\hbar\omega_{mn} \equiv E_m - E_n \to \omega_{ij} + \omega_{jk} = \omega_{ik}$.
Kramers, van Vleck, Born (inspired by Bohr's correspondence principle in order to describe dispersion) $d/dJ \to (1/\hbar)\Delta/\Delta n$. By applying this to Bohr-Sommerfeld, Heisenberg obtained what Born finally interpreted as $[q, p] = i\hbar$. Also, from Rydberg-Ritz-Bohr's combination principle $\omega_{ij} + \omega_{jk} = \omega_{ik}$ and Heisenberg's guess $x(t) = \sum_{mn} x_{mn} \exp(i\omega_{mn} t)$, Born (and Jordan) deduced that x_{mn} are matrices, and Dirac found a map between Posson brackets and commutators - Hamiltonian picture [Schrödinger - Hamilton-Jacobi picture; Dirac-Feynman-Schwinger: Lagrangian (covariant) picture]

Quantum theory (together with relativity) is crucial for the existing (albeit, incomplete) structure of **simple, as opposed to complex** physics defined by 3 fundamental constants (leading to Planck's fundamental length $l_P = (G\hbar/c^3)^{1/2}$ and time $t_P = l_P/c$, etc):
c - **velocity of light** (conversion of space to time; "measures how fast quantum information is transferred in quantum spacetime")
G - **gravitational constant** ("elasticity" of spacetime, measures how geometry reacts to matter; "measure of the curvature of general geometry of quantum probability")
$\hbar$ - **Planck's constant** (measure of "virtuality" of histories of a physical system; "conversion of general quantum probability to time at Planck energy")
6 known combinations: origin = ($c \to 0, G \to 0, \hbar \to 0$), c, G, Gc, $\hbar$, $\hbar c$ (classical spacetime, quantum matter)
2 unknowns: $\hbar G$, $cg\hbar$ (quantum spacetime, quantum matter)

Classical spacetime - story of symmetrization:

Aristotle: space and time separate (preferred frames; absence of inertia)

Galileo: space mixes with time, but time not with space (absolute time) - inertial frames and relativity

Newton: curved (but locally Galilean) space with absolute time - gravity as geometry

Minkowski: space and time symmetrized (space mixes with time and time with space; observer dependent space and time as different "slices" of spacetime); $ds_M^2 = \eta_{ab} dx^a dx^b$.

Einstein: curved spacetime, dynamical causality (symmetrization of Newton); gravity as geometry of spacetime; $ds_E^2 = g_{ab}(x) dx^a dx^b$.

Newtonian physics as the geometric Newton-Cartan formulation. (Deeper theory (Einstein's) sheds light on its limits (Newton's), conceptually and practically. Correspondence principle.)

Quantum theory: story of symmetrization **(Koopman-von Neumann viewpoint)** (also in Schrödinger's picture):
Classical physics in terms of non-commuting objects:
$[q, p] = 0$ but $[q(p), \partial/\partial q(p)] = -1$ (operators needed to write classical equations of motion $\dot{p} = -\partial H/\partial q$ and $\dot{q} = \partial H/\partial p$)
In case of classical statistical physics, probability emergent and Kolmogorov-like - additive; no interferences!
Upon symmetrization (as in the context of spacetime physics) we get quantum physics: $[q, p] = i\hbar$ and $[q(p), \partial/\partial q(p)] = -1$
Probability measure maximally symmetric (Fisher) and consistent with equations of motion. Intrinsic probability. and not Komogorov-like. Quadratic Born rule $|\psi|^2$ (or Gleason for density matrices, ρ, $Tr\rho = 1$, so that $\langle O \rangle \equiv Tr(\rho O)$, in general). Interference of probabilities. But no triple or higher order interference!
Similarly for fields, like EM, $q \to \vec{A}(x)$!

Note: **Space time physics** (gravity) is classical and it has **fixed polarization** (spacetime!), but it has **dynamical geometry**.
Quantum physics (matter) is intrinsically probabilistic and it has **dynamical polarization** $\psi \to U\psi$ (U - unitary) so that $|\psi|^2$ does not change, but the **geometry of quantum theory is fixed** (complex projective spaces CP^n, the simplest being the Bloch sphere, the Schrödinger equation being the geodesic equation on CP^n, and the Born rule being the Fubini-Study distance).
So, quantum theory of both spacetime and matter (also known as quantum gravity) should have dynamical polarization and dynamical geometry and it should be intrinsically probabilistic. (Probability is contextual!)
The missing physics of $c\hbar G$ and $\hbar G$: **Quantum Gravity (QG) = Gravitized Quantum Theory (GQ) (Dynamical and contextual Born rule! Quanta of spacetime!)** (Note: classical general relativity "gravitizes" (covariantizes) all of classical physics!)

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QG=GQ

What would be the first experimental consequence of "dynamical Born rule" or "gravitized quantum theory"? (Beglund, Geraci, Hübsch, Mattingly, Minic)

Triple (triple slit!) and higher-order quantum interference! (Experiment possible in the next few years.)

Let's repeat: canonical quantum theory does not have intrinsic triple quantum interference (consequence of the Born rule and the fixed geometry of the complex projective space). **In quantum theory: triple interference is a measurement of zero.**

Current experimental bounds (photonic) - rather weak (10^{-3})*.* *Neutrino bounds expected to be surprisingly similar (and to be measured at JUNO).*(Huber, Minakata, Minic, Pestes, Takeuchi) and (Bhatta, Minic, Takeuchi). **No tests of the Born rule with gravity!**

QG=GQ

In more detail (Sorkin): Classically, we have addition of probabilities (Kolmogorov)

$$P_n(A, B, C, \cdots) = P_1(A) + P_1(B) + P_1(C) + \cdots \ , \qquad (1)$$

for any number of paths. Quantum mechanically, we have for two paths $P_2(A, B) = |\psi_A + \psi_B|^2$ or more explicitly

$$|\psi_A|^2 + |\psi_B|^2 + (\psi_A^* \psi_B + \psi_B^* \psi_A) \equiv P_1(A) + P_1(B) + I_2(A, B) \quad (2)$$

where the last term

$$I_2(A, B) = P_2(A, B) - P_1(A) - P_1(B) \qquad (3)$$

is the "interference" of the two paths A and B. **Non-vanishing double-path interference, $I_2(A, B) \neq 0$, distinguishes quantum theory from the classical one.**

QG=GQ

The **Born rule** dictates that all the superimposed paths only interfere with each other in a pairwise manner. For instance, for three paths we have $P_3(A, B, C) = |\psi_A + \psi_B + \psi_C|^2$

$$P_2(A, B) + P_2(B, C) + P_2(C, A) - P_1(A) - P_1(B) - P_1(C), \qquad (4)$$

where only pairwise interferences between the pairs (A, B), (B, C), and (C, A) appear.
It is clear from the above that in order for there to be a triple and higher order correction in an interference pattern the Born rule must be relaxed. **Note: Born rule is the "entanglement witness" in the EPR-Bell experiment. [Entanglement of masses (QGEM experiment)].**

Consider **a triple slit experiment**: Since only pairwise interferences between the pairs (A, B), (B, C), and (C, A) appear, it makes sense to define any deviation from this relation as the intrinsic triple-path interference $I_3(A, B, C)$ (Sorkin)

$$P_3(A,B,C)-P_2(A,B)-P_2(B,C)-P_2(C,A)+P_1(A)+P_1(B)+P_1(C). \tag{5}$$

(This can be easily generalized for the case of n-paths.) For both classical and quantum theory, this intrinsic triple-path interference is zero for any triplet of paths. **Experimental confirmation of $I_3 = 0$ would be a confirmation of the Born rule.**
Weak bounds were placed on the parameter ($\kappa \sim 10^{-3}$) in photonic experiments (**Again: no check of the Born rule with gravity exists!**)

$$\kappa = \frac{\varepsilon}{\delta}, \quad \varepsilon = I_3(A,B,C), \quad \delta = |I_2(A,B)| + |I_2(B,C)| + |I_2(C,A)|. \tag{6}$$

The claim (Berglund, Geraci, Hubsch, Mattingly, Minic) is that **with quantum gravitational degrees of freedom turned on, one can get $I_3 \neq 0$,** but for that one needs **gravitized quantum theory**, with **observer dependent spacetime/Hilbert spaces and dynamical Born rule**. The generalized probability in this approach to quantum gravity is given by (analogy with non-linear optics), which, geometrically, looks very Finsler-like

$$P = g_{ab}(\psi)\,\psi_a\psi_b \equiv \delta_{ab}\,\psi_a\psi_b + \gamma_{abc}\,\psi_a\psi_b\psi_c + \ldots, \qquad (7)$$

where a, b, c are state-space indices and with (schematically) - Schrödinger plus Nambu quantum theory (DM, Tze) (DM, Takeuchi, Tze) (Bhatta, DM, Takeuchi) ($\hbar G$)

$$\frac{\mathrm{d}\psi_a}{\mathrm{d}\tau} = E_{ab}\psi_b + \Gamma_{abc}\,\psi_b\psi_c, \qquad (8)$$

where τ is an evolution parameter. **There exists a nonlinear optics analog experiment! (Vienna group)**

Effective triple interference - possible in non-linear optical media! (Instead of ψ, non-linear waves; instead of probability P - non-linear/cubic energy density.) **"Smoking gun": Talbot effect on a diffraction grating $\to$ non-linear Talbot effect.** *Effect of decoherence different from the observable signatures of the non-linear Talbot effect.* **Thus intrinsic triple interference with quantum gravity degrees of freedom - analogous to a non-linear "quantum spacetime medium".** (*Note: non-linear quantum theory with fixed Hilbert spaces is NOT GQ!*) **Also: Need to go beyond traditional generalized probability theories (GPT) - non-Markovian nature of gravity due to its memory!** Computation of the vacuum energy/cosmological constant (below), suggests a low energy scales: $10^{-3}eV$ or $10^{-4}m$.

"Bottom-up" reason for "gravitization of quantum theory". (Minic, Tze) **Geometry of quantum theory** (review by Ashtekar and Schilling) - maximally symmetric geometry of CP^N, with Fubini-Study (FS) metric (maximally symmetric Fisher metric of information theory (Wootters)) $ds_F^2 = \sum_i \frac{dp_i^2}{p_i}, \sum_i p_i = 1$ where p_i are probabilities. Let $p_i \equiv x_i^2$ so that we have a sphere in a Euclidean space $ds_F^2 = \sum_i dx_i^2, \sum_i x_i^2 = 1$. If $i = 1, ..., 2(N+1)$, (x_is real and imaginary components of the wavefunction), then we have an odd-dimensional sphere S^{2N+1}, which is always a $U(1)$ bundle of CP^N (with FS metric). **Compatibility of Fisher with the symplectic form - equations of motion - gives the complex structure, that is i in the Schrödinger equation.**

Schrödinger equation is the geodesic equation on CP^n.
Born rule - the FS distance on CP^n: $ds^2_{FS}(1,2) = 4(1 - |\langle\psi_1|\psi_2\rangle|^2)$
Note $CP^n = U(n+1)/U(n) \times U(1)$; $U(n+1)$ - unitary evolution; $U(n)$ - non-Abelian Berry phase; $U(1)$ - the complex phase of the wave function; Entanglement - Segre embedding of the product of lower dimensional CP^ns into a higher dimensional CP^N. (*Density matrices - Bures metric - analog of FS/Fisher*). From the Schrödinger equation derive (Aharonov and Anandan)

$$2\hbar ds_{FS} = \Delta E dt, \tag{9}$$

where ΔE is the dispersion of energy, and ds_{FS} is the Fubini-Study metric on CP^n (which is directly related to the Fisher metric).
Quantum spacetimes (topology change) - no unique timelike Killing vector - and thus ΔE is state dependent, so the geometry is state dependent, and therefore, dynamical (or $\Delta E \to E_P$). (Minic, Tze).
Thus: Generally covariant quantum theory with a dynamical Born rule.

How do we formulate $QG = GQ$? Need a deeper understanding of quantum theory and spacetime. **Derivation of quantum theory from fundamental length/time and Lorentz symmetry of spacetime:** To reconcile the two, need observer dependent spacetimes in superposition. Lattice of spacetime, and self-dual lattice of phase space. Intrinsic probability, homogeneous and isotropic (Fisher). Intersection of orthogonal group of probability and symplectic group of phase space - unitarity. Born rule. On the lattice **modular variables** a la Aharonov et al, that commute (!)

$$[\hat{q}]_a \equiv \hat{q} \bmod a \quad [\hat{p}]_{\frac{2\pi\hbar}{a}} \equiv \hat{p} \bmod \frac{2\pi\hbar}{a} \quad \implies [[\hat{q}]_a, [\hat{p}]_{\frac{2\pi\hbar}{a}}] = 0 \tag{10}$$

but imply $[q, p] = i\hbar$ (!!), via the generators of translations in **phase space**

$$\hat{U}_a = e^{\frac{i}{\hbar}\hat{p}a}, \quad \hat{V}_{\frac{2\pi\hbar}{a}} = e^{\frac{i}{\hbar}\hat{q}\frac{2\pi\hbar}{a}}, \quad \implies [\hat{U}_a, \hat{V}_{\frac{2\pi\hbar}{a}}] = 0 \tag{11}$$

QG=Gravitization of Quantum Theory

Modular variables are covariant (modular energy, modular time as well).

Take fundamental length λ and energy ϵ, so that $\lambda\epsilon \equiv \hbar$. **Modular variables are non-local (but consistent with causality - origin of the uncertainty principle)**.

Contextuality: *in a double slit experiment the parameters* λ *and* ϵ *are contextual to the experiment*.

Note that in the above derivation of quantum theory, if we let spacetime be dynamical, we get a "gravitization of quantum theory" with a dynamical Born rule and general transformations of states, not just unitary! Also the measure of quantum probability becomes contextual!

QG=Gravitization of Quantum Theory

Explicit non-locality: Take $H = \frac{p^2}{2m} + V(q)$ and write the Heisenberg equation of motion for $e^{ipR/\hbar}$, or equivalently $[p]_R$ (R - contextuality parameter, such as the distance between two slits in the double slit interference experiment). (Aharonov et al)

$$\frac{d[p]_R}{dt} = \ldots \frac{V(q+R/2) - V(q-R/2)}{R} \tag{12}$$

Quantum mechanics=non-locality (from contextual modular variables) plus causality (compatibility with Lorentz). Now, reformulate quantum mechanics (QM) using (covariant) modular variables via modular spacetime. (**Note: Quantum theory tells us something new about quantum spacetime!**)

QG=Gravitization of Quantum Theory

Introduce **modular spacetime**. What is **modular space**? (FLM, '16)
Modular space is the space of all commuting subalgebras of the Heisenberg-Weyl algebra.
Note $[q, p] = i\hbar$ - Heisenberg-Weyl algebra, whereas $[[q]_a, [p]_{2\pi\hbar/a}] = 0$ - commuting subalgebra of Weyl-Heisenberg.
Mackey's theorem: **The space of all commuting subalgebras of the Heisenberg-Weyl algebra is a self-dual phase space lattice lifted to Heisenberg-Weyl.** (We have seen this already!)
Use covariant modular variables - **modular spacetime** of d spacetime dimensions.

Note (FLM): **phase space - symplectic structure** $Sp(2d)$ - ω_{ab}.
Self-dual lattice (l plus $\tilde{l}$) - **doubly-orthogonal** $O(d, d)$ - η_{ab}.
To **define the vacuum** on this self-dual lattice - **need doubly metric structure** $O(2, 2d-2)$ - H_{ab}.
ω, η, H **define Born geometry.** (FLM, '13, '14, '15, '16)
Their triple intersection gives the Lorentz group.
Thus **QM (special quantum relativity) follows from non-locality (fundamental length/time) consistent with causality. (In general: dynamical causality and geometry - gravitized quantum theory (general quantum relativity).)**
Note: can be localized (local QFT possible!) in a particular phase space cell, but can't tell in which one (uncertainty principle)!

QG=Gravitization of Quantum Theory

How can fundamental length/time be consistent with Lorentz?
This is possible because of **relative (observer dependent) locality**. (Amelino-Camelia, Freidel, Kowalski-Glikman, Smolin) Different observers see different spacetimes (slices of modular spacetime). **Different spacetimes are in linear superposition, and so fundamental length/time is consistent with Lorentz.** (Similar to spin: the superposition of up and down spin gives the Bloch sphere which is consistent with rotation symmetry, even though spin is discrete.) (FLM)

Most important: **Gravity violates assumptions of Chentsov theorem about the uniqueness of the Fisher metric (which is Euclidean!) from sufficient statistics (BGHMM, '25)**
Chentsov theorem: *Fisher metric is the unique information metric under sufficient statistics and identical independent measurements*
(**1**) Data is a set of permanent results from independent identically distributed (i.i.d.) measurements - in quantum gravity the data is not i.i.d. but instead non-Markovian, as each successive recording of a data point affects the next measurement (due to equivalence principle; to store data requires energy which is a gravitational charge and thus back-reacts). and (**2**) sufficient statistics cannot be generated and passed between observers in quantum gravity due to different boundary conditions (gravity cannot be screened). **As a result, the question of whether the Born rule is fixed or dynamical in quantum gravity becomes an empirical question.**

QG=GQ

Generic quantum polarization (FLM, '16) - **modular polarization** (defined via the Zak transform). Given Schrödinger's $\psi_n(x)$

$$\psi_\lambda(x, \tilde{x}) \equiv \sqrt{\lambda} \sum_n e^{-2\pi i n \tilde{x}} \psi_n(\lambda(n + x)) \tag{13}$$

($x \equiv q/\lambda$, $\tilde{x} \equiv p/\epsilon$, so $[x, \tilde{x}] = i$, $\lambda\epsilon = \hbar$). Note, from the point of view of modular polarization, Schrödinger's polarization is very singular. Introduce $\mathbb{X}^A \equiv (x^a, \tilde{x}_a)^T$, so that $[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b] = i\omega^{AB}$. We can write the translations operators in phase space covariantly $W_{\mathbb{K}} \equiv e^{2\pi i \omega(\mathbb{K}, \mathbb{X})}$, where $\mathbb{K}$ stands for the pair $(\tilde{k}, k)$ and $\omega(\mathbb{K}, \mathbb{K}') = k \cdot \tilde{k}' - \tilde{k} \cdot k'$. (Note W - **Aharonov-Bohm phases - prototypical example of modular variables**.)

QG=GQ

So far we have discussed covariant quantum phase space as an example of modular space, and so we are ready to discuss modular (quantum) spacetime. Consider (FKLM, '18) a **metaparticle** (mp) propagating in a modular space defined by Born geometry - ω, η, H. The metaparticle world-line action $S_{mp} = \int d\tau L_{mp}$ (canonical particle - $\mu \to 0$ and $\tilde{p} \to 0$)

$$L_{mp} = p_\mu \, \dot{x}^\mu + \tilde{p}^\mu \, \dot{\tilde{x}}_\mu + \lambda^2 p_\mu \, \dot{\tilde{p}}^\mu - \frac{N}{2} \left(p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu - m^2 \right) + \tilde{N} \left(p_\mu \tilde{p}^\mu - \mu \right), \tag{14}$$

where ω is in ("Berry-phase") $p_\mu \, \dot{\tilde{p}}^\mu$, and η in the diffeo constraint $p_\mu \tilde{p}^\mu = \mu$ and H in the Hamiltonian constraint $p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu = m^2$. **Dual spacetime** $\tilde{x}$, $[x, \tilde{x}] = i\lambda^2$, and **dual momentum space** $\tilde{p}$, $[p, \tilde{p}] = 0$. (Also, $[x, p] = i\hbar = [\tilde{x}, \tilde{p}]$.) **Spacetime** x **is quantum!**

QG=GQ

The metaparticle can be understood also as follows: If one second quantizes Schrödinger's $\psi(x)$ one naturally ends up with a quantum field operator $\hat{\phi}(x)$. Similarly, the second quantization of the modular $\psi_\lambda(x, \tilde{x})$ would lead to a modular quantum field operator $\hat{\phi}_\lambda(x, \tilde{x})$ (**modular fields - metafields**)

$$\hat{\phi}(x) \to \hat{\phi}_\lambda(x, \tilde{x}). \tag{15}$$

with $[x, \tilde{x}] = i\lambda^2$ - covariant non-commutative field theory. (FLM, '17) **Mixing of UV and IR. Not EFT. Contextual. Also, in CMP (Barnes, Heremans, DM).** Classical spacetime label x of canonical QFT - choice of (classical spacetime) polarization in modular (quantum) spacetime with a contextuality parameter λ. **Average over dual spacetime $\tilde{x}$ to obtain classical evolution from the unitary evolution in the spacetime basis!**

QG=GQ

Quanta of canonical quantum fields $\phi(x)$ - particles (and their antiparticles).
Quanta of modular quantum fields $\phi_\lambda(x, \tilde{x})$ - metaparticles.
First prediction of modular spacetime approach to quantum theory - metaparticles! (FKLM) (We will argue that dual particles, correlated to visible particles, represent dark matter.)
[If $\vec{\tilde{p}} = 0$, then $E_p^2 + \frac{\mu^2}{E_p^2} = \vec{p}^2 + m^2$.]
Note that if we turn on backgrounds $p \to p + \phi$ and $\tilde{p} \to \tilde{p} + \tilde{\phi}$. Thus we have **"dark matter" fields** $\tilde{\phi}(x)$ in the effective classical spacetime x description (after integrating over the dual spacetime $\tilde{x}$ - quantum measurement). **Visible ϕ and Invisible (dark matter) $\tilde{\phi}$ do not commute - $\tilde{\phi}$ describes fuzzy dark matter.**

QG=GQ

Explicit realization in terms of a chiral phase-space reformulation of the bosonic string, the "**metastring**," (FLM, '13, '14, '15) - also a non-perturbative proposal (BHM '21, '22) of QG (**matrix model-like**, time-asymmetric (?), $\partial_\sigma \cdot \equiv [\hat{\mathbb{X}}, \cdot]$, where $\hat{\mathbb{X}}$ matrix comes from modular world-sheet) - spacetime/matter quanta

$$S_{\rm str}^{\rm ch} = \int \mathrm{d}\tau \mathrm{d}\sigma \left[\partial_\tau \mathbb{X}^a \big(\eta_{ab}(\mathbb{X}) + \omega_{ab}(\mathbb{X})\big) - \partial_\sigma \mathbb{X}^a H_{ab}(\mathbb{X})\right] \partial_\sigma \mathbb{X}^b, \tag{16}$$

where $\mathbb{X}^a \equiv (X^a/\lambda, \tilde{X}_a/\lambda)^T$ are coordinates on phase-space like (doubled) target spacetime and η, H, ω are all dynamical. $x^a, \tilde{x}_a$ come from the left and right moving modes of the bosonic string,

$$x^a \equiv x_L^a + x_R^a, \quad \tilde{x}^a \equiv \tilde{x}_L^a - \tilde{x}_R^a \tag{17}$$

In the context of a flat metastring we have constant η_{ab}, H_{ab} and ω_{ab} (zero ω_{ab} - connection to double field theory)

$$\eta_{ab} = \begin{pmatrix} 0 & \delta \\ \delta^T & 0 \end{pmatrix}, \quad H_{ab} = \begin{pmatrix} h & 0 \\ 0 & h^{-1} \end{pmatrix}, \quad \omega_{ab} = \begin{pmatrix} 0 & \delta \\ -\delta^T & 0 \end{pmatrix}, \tag{18}$$

The standard Polyakov action is obtained when setting $\omega_{ab} = 0$ and integrating out the $\tilde{x}_a$ ($h_{ab}(X)$ - gravity; zero beta function for h_{ab} - Einstein's equations)

$$S_P = \int \mathrm{d}\tau \mathrm{d}\sigma \gamma^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b h_{ab}(X) + \dots \tag{19}$$

The triplet (ω, η, H) define the Born geometry (FLM, '13, '14) and the metastring propagates in modular, not classical, spacetime. Recall: the *space* of commuting subalgebras of the Heisenberg algebra, $[\hat{x}, \hat{\tilde{x}}] = i\lambda^2$, defines modular spacetime (FLM, '15, '16). **Metastring =** $c\hbar G$.

The new feature in the metastring formulation of the bosonic string is intrinsic non-commutativity - a new Heisenberg algebra (vertex operators $V_{\mathbb{K}} \equiv e^{2\pi i \mathbb{K}\cdot\mathbb{X}}$ reps of Weyl-Heisenberg - no cycocles)

$$[\mathbb{X}^a, \mathbb{X}^b] = i l_s^2 \omega^{ab} \implies [X^a, \tilde{X}^b] = i\delta^{ab} l_s^2 \quad (20)$$

$[\delta q \sim G_N \delta p \to \delta q \sim G_N \delta \tilde{p} \sim G_N \frac{1}{\delta \tilde{q}} \to \delta q \delta \tilde{q} \sim G_N \to [q, \tilde{q}] \sim i l_P^2]$ as well as the standard commutators (with $[\Pi, \tilde{\Pi}] = 0$)

$$[\mathbb{X}^a, \mathbb{P}_b] = i\hbar\delta^a_b \implies [X^a, \Pi_b] = i\delta^a_b \hbar, \quad [\tilde{X}^a, \tilde{\Pi}_b] = i\delta^a_b \hbar \quad (21)$$

Note, if Kalb-Ramond B_{ab} (axion) constant but non-zero, dual coordinates do not commute! In general - non-associativity (FLM,'17.) **[SM group from phases of unique octonionic quantum geometry** $F_4/SO(9)$ - **Gursey, Gunaydin]** Also: **Zero modes of the metastring - metaparticles** (FKLM, '18, '21) - little rigid strings. (Each Standard Model (SM) particle has a correlated "dual" - dark matter (BHM, '20, '21).)

QG=GQ

QG=GQ - **Analogy with relativity**
Quantum Relativity in analogy with **Classical Relativity**
Classical relativity:
a) special relativity - motivated by EM - (*Minkowski spacetime/geometry, relativity of simultaneity*),
b) relativistic field theory (*reps of Lorentz/Poincare, particles/antiparticles*),
c) general relativity (*dynamical classical spacetime*)
Spacetime relativity - first (classical) relativity (both spacetime and matter classical).

QG=GQ

Quantum relativity: (FLM, '13, '14, '15, '16, '17)
A) QM from quantum spacetime (**modular spacetime, Born geometry, relative locality**),
B) QFT (**metafields/metaparticles**),
C) **gravitized quantum theory** (**dynamical quantum spacetime**, **dynamical Born geometry**, **metastrings**, metaparticles - dark matter, geometry of dual spacetime - dark energy)
QM/QFT - second (quantum) relativity (matter quantum, spacetime classical).
QG=GQ - third (QG) relativity (spacetime/matter quantum).
(Third relativity - Finkelstein; Wheeler) **Dynamical and contextual quantum probabilities. Background independent quantum theory.**

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QG=GQ and Experiment:

Vsible and invisible matter and energy in classical spacetime x:

$$S_{eff} = -\frac{1}{G}\int \sqrt{-g(x)}[R(x) + \Lambda + L_m(A_m(x)) + L_{dm}(A_{dm}(x))], \tag{22}$$

Λ is the cosmological constant and the simplest way to account for the observed accelerated expansion of the universe. Λ/G corresponds to the vacuum (dark) energy (cosmological constant problem - why it is small and positive, of the order of $(10^{-3}eV)^4$). **Λ, viewed as vacuum energy, is the first observed quantum gravity phenomenon**. DM denotes invisible (dark) matter A_{dm}. Visible matter $A_m(x)$ is described by the Standard Model with 25 particles (fermions: six quarks, six leptons; bosons: photon, $W^{\pm}$, Z, eight gluons, and the Higgs boson), with 26 free parameters.

QG=GQ and Experiment:

Let's start with dark matter. Dual "particles" (dual fields) - dark matter (derive from the metastring to leading order in λ)

$$S_{eff} = -\int \sqrt{g(x)\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x}) + L_m(A(x,\tilde{x})) + \tilde{L}_{dm}(\tilde{A}(x,\tilde{x}))], \tag{23}$$

Here the A fields denote the usual Standard Model fields, and the $\tilde{A}$ are their duals, as predicted by the general (modular) formulation of quantum theory that is sensitive to the minimal length.
Note that we need to integrate over the dual space coordinates $\tilde{x}$ to get an effective description of **visible matter,** $A(x)$**, and dark (fuzzy) matter,** $\tilde{A}(x)$, in classical x spacetime. (BHM, '21, '22)

QG=GQ and Experiment:

Dynamical geometry of dual spacetime - dark energy (to leading order in λ) [model for the equation of state of dark energy]

$$S_{eff} = -\int \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x}) + ...], \qquad (24)$$

In this leading limit, the $\tilde{x}$-integration in the first term defines the gravitational constant G_N, and in the second term produces a **positive cosmological constant constant** Λ! (BHM, '21, '22) **In general: time-varying dark energy (Hur, Jejjala, Kavic, Minic, Takeuchi, '25)** - DESI! *Visible and dark matter correlated (via the minimal length* λ*) - fuzzy dark matter. Fuzzy dark matter is quantum and knows about* Λ *- Milgrom's scaling (galaxies, clusters, superclusters) and acceleration* $a_0 \sim cH/(2\pi)$. ($\Lambda \sim H^2$.) (Edmonds, Erlich, Hur, Minic, Takeuchi).

QG = GQ and Experiment:

The computation of the cosmological constant (Λ)/vacuum energy (FKLM, '22): $Z_{vac} = \langle 0 | e^{-iH\tau} | 0 \rangle = e^{-i\rho_0 V_D}$ where V_D is the volume of D-dimensional spacetime, and ρ_o is the vacuum energy density. Particles (p): $Z_{vac} = exp(Z_{S^1}) \equiv exp(Z_p)$; Strings (s): $Z_{vac} = exp(Z_{T^2}) \equiv exp(Z_s)$. So, $\rho_0 \sim Z_{p,s}/V_D \sim \Lambda^D$ ($V_D \sim l^D$) In both cases: $Z_{p,s} \sim V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} \cdots \sim l^D \Lambda^D$. *Modular regularization of phase space volume:* $l^D \Lambda^D = N$. Holography: $N \sim l^{D-2}/l_P^{D-2}$. In $D = 4$, $\rho_0 \sim 1/(l^2 l_P^2)$ and thus $m_\Lambda \sim \sqrt{M M_P} \sim 10^{-3} eV$. ($M$ - Hubble scale; M_P - Planck scale.) $M = 10^{-34} eV$; $M_P = 10^{19} GeV$. Note, M and M_P are **contextual** *IR* and *UV* scales. **Implied by dynamical phase space (because of dynamical spacetime!) and thus, dynamical quantum phase space. Thus, dynamical Born rule. QG=GQ.**

QG = GQ and Experiment:

Repeat the same logic for some *effective action* **that leads to the masses of elementary particles (BHM, '23).** First, the Higgs - $M_H^2 \sim g_s^2 M_s^2$, where the relevant *IR* scale is the **Bjorken-Zeldovich scale (BZ),** M_{BZ}**. Match spacetime and matter entropies**: $N \sim l^2/l_P^2$ **to** l^3/l_{BZ}^3, so that $l_{BZ}^3 \sim l l_P^2$. ($M_{BZ}^3 \sim M M_P^2 \sim (7MeV)^3$). The UV scale is M_P. Obtain, $m_H \sim \sqrt{m_\Lambda M_P}$. (Here we use some stringy formulae for m_H.) Repeat for the Standard Model (SM) fermions: $m_f \sim g_s M_s$ and use the phase-space-like structure of modular spacetime. The relevant IR scale - M_{BZ} and the relevant UV scale - the heaviest fermion scale. (Use SM criticality to relate the heaviest fermion masses to the Higgs mass.) Two expressions are possible:
$m_f \sim M_{IR}\sqrt{\frac{M_{UV}}{M_{IR}}} \sim \sqrt{M_{IR} M_{UV}}$ or $m_f \sim M_{IR}\sqrt{\frac{M_{IR}}{M_{UV}}}$

QG = GQ and Experiment - Results 1:

CC: $m_\Lambda \sim \sqrt{MM_P} \sim 10^{-3}eV$; $m_H \sim \sqrt{m_\Lambda M_P} \sim 125GeV$; m_H and RG: Determine m_t, m_b and m_τ from m_H (SM criticality). $M_{BZ}^3 \sim MM_P^2 \sim (7MeV)^3$. **Neutrinos**: $l^2/l_P^2 \sim l^4/l_\nu^4 \to l_\nu \sim l_\Lambda$!

So: $m_c \sim \sqrt{M_{BZ}\, m_t} = M_{BZ}\sqrt{\frac{m_t}{M_{BZ}}} \sim 1.10\ (1.27)\,\text{GeV}$.

$m_s \sim \sqrt{M_{BZ}\, m_b} = M_{BZ}\sqrt{\frac{m_b}{M_{BZ}}} \sim 171\ (93.4)\text{MeV}$.

$m_u \sim M_{BZ}^2/m_c \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_t}} \sim 10^{-2}M_{BZ} \sim 10^{-1}\ (2.16)\text{MeV}$.

$m_d \sim M_{BZ}^2/m_s \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_b}} \sim 10^{-1}M_{BZ} \sim 1\ (4.67)\text{MeV}$.

$m_\mu \sim \sqrt{M_{BZ}\, m_\tau} = M_{BZ}\sqrt{\frac{m_\tau}{M_{BZ}}} \sim 112\ (106)\text{MeV}$.

$m_e \sim \frac{M_{BZ}^2}{m_\mu} \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_\tau}} \sim 464\ (511)\text{keV}$. (**Neutrino masses:**)

$m_3 \sim m_H^2/M_{SM} \sim (10^{-2})\text{eV}$, $m_2 \sim \sqrt{m_\Lambda m_3} \sim (10^{-3})\text{eV}$.

$m_1 \sim \frac{{m_\Lambda}^2}{m_2} \sim (10^{-4})\text{eV}$.

QG = GQ and Experiment - Results 2:

CKM matrix (quark mixing matrix)

$$|V_{cb}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_d}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_d}} \sim 0.050 \quad (0.041), (\rightsquigarrow \theta_{23})$$

$$|V_{td}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_s}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_s}} \sim 0.011 \quad (0.008)(\rightsquigarrow \theta_{12})$$

$$|V_{ub}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_b}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_b}} \sim 0.002 \quad (0.003)(\rightsquigarrow \theta_{13})$$

PMNS (neutrino mixing matrix: $M_{BZ} \to m_\Lambda$)

$$|U_{\mu 3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_1}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_1}} \sim 0.50, \quad (0.63)$$

$$|U_{\tau 1}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_2}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_2}} \sim 0.13, \quad (0.26)$$

$$|U_{e3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_3}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_3}} \sim 0.06, \quad (0.14) \text{ (Similar structure!)}$$

Coincidence? 20 parameters written in terms of Hubble and Planck scales - QG - (and the Standard Model scale)!

QG=GQ - Explicit Realization

Finally: *Matter is granular and cuttable*: it consists of fermions that are held together through interactions that are mediated by bosons (the spin-statistics theorem of local QFT).
Note the fundamental difference between spacetime and matter: *spacetime is extended and non-cuttable*. **(Spacetime atoms, "ameres", are different than matter atoms.)** $(\Delta N^2 \sim N)$
The claim here is that **spacetime quanta obey infinite statistics** (*quantum distinguishable, or quantum Boltzmann statistics,* $a_i a_j^\dagger = \delta_{ij}$ - Cuntz free algebra and non-commutative probability theory applicable to matrix models) and are *held together by higher order quantum correlations responsible for higher order interference effects.* (Spacetime Avogadro number $N \sim 10^{31}$ and $l_\Lambda \sim \sqrt{l l_P}$.) **Relational spacetime. Gravitational Brownian motion via gravitational interferometry.** (Zurek et al).

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Summary

Quantum theory = quantum spacetime plus Lorentz.
Quantum gravity (QG) = Gravitization of quantum theory (GQ)
Gravitization of quantum theory = dynamical geometry of quantum theory (dynamical Born geometry). Not only double, but also triple and higher order interference allowed! "Smoking gun experiment". Quantum spacetime = modular spacetime (geometry of quantum theory). Spacetime and its dual do not commute! (Measurement!) Consequences: Particles (visible matter) are singular limits of metaparticles - correlated particles and dual (dark) particles. Quantum fields and dual (dark) quantum fields do not commute (fuzzy dark matter). Dark energy - curvature of dual spacetime. Metaparticles - zero modes of the metastring (non-commutative string in a dynamical geometry of modular spacetime) (QG=GQ) New view on quantum fields! (CMP, HEP...)

Quantum theory teaches us about: 1) quantum spacetime, 2) new probability! **a) Quantum measurement and dual spacetime**: Dual spacetime labels can be viewed as covariant, non-local, and non-commutative hidden variables. Upon averaging over dual variables (to leading order in the fundamental length) can show that classical equations follow. *(Classicalization plus decoherence.)* **b) Complexity and mock quantum theory (Hübsch, Nikolic, Pajevic, DM)**: Should Kolmogorov probability be used in generic stochastic non-equilibirum complex adaptive systems? If not (violation of Chentsov's theorem in complex adaptive systems!) then one can show (at least in theory) that **emergent, or analog, or "mock" quantum theory** (and its dynamical, "gravitized" version) is a universal description. (Reason for Nelson's stochastic approach or de-Broglie-Bohm?) *[Use quantum description of classical systems (Koopman-Von-Neumann in Schrödinger's picture).]* (Bolmatov, DM)

Outlook

Phenomenological Implications of QG=GQ and future work:

- Cosmological constant (CC) as a guiding empirical quantum gravity phenomenon - (also, gravitational interferometry and QG Brownian motion; LHC signals - 2d reduction)
- Metaparticles (zero modes of the metastring) and dark matter (entangled/correlated SM/dual (DM) particles) - fuzzy DM
- Dark energy (CC plus time variation) as the curvature of the dual spacetime (CC naturally small) - DESI, etc.
- Gravitational wave "echoes" - $N \sim l^2/l_P^2$ for black holes (quantum chaos - quantum scars?).
- Dynamical Born rule, "QG=GQ" - triple (and higher order) quantum interference in QG! Contextuality of probability!
- The CC, the Higgs mass and SM fermion masses and mixing. (**Neutrino masses! Bounds - Cosmological Attractor?**)